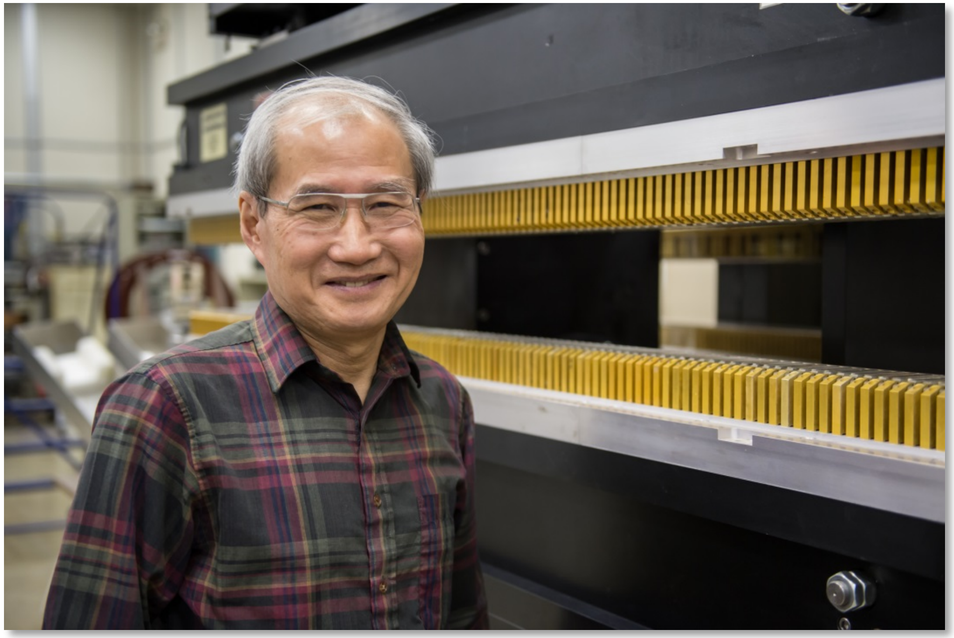
SLAC NATIONAL ACCELERATOR LABORATORY

FEL theory: From LEUTL to LCLS

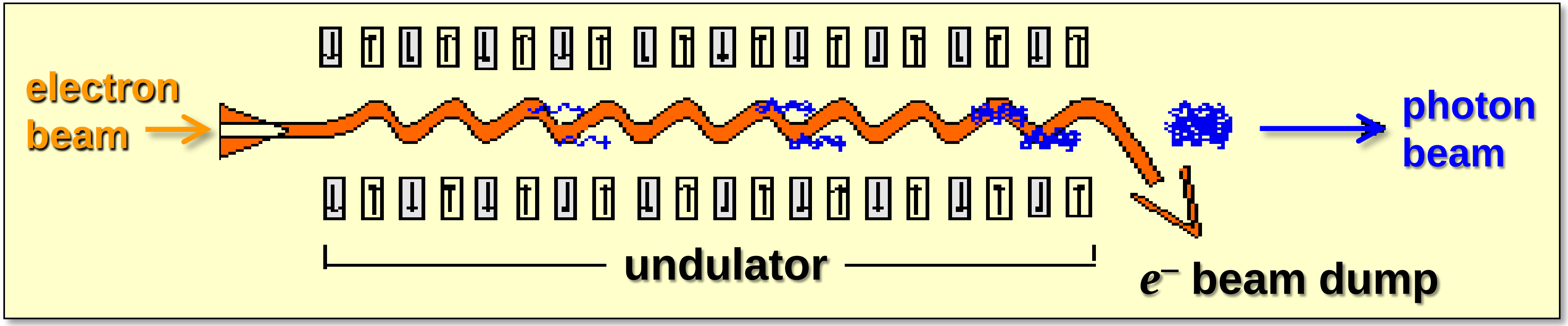
Zhirong Huang (SLAC)
March 15, 2019

Coherence in particle and photon beams:
Past, Present, and Future Symposium

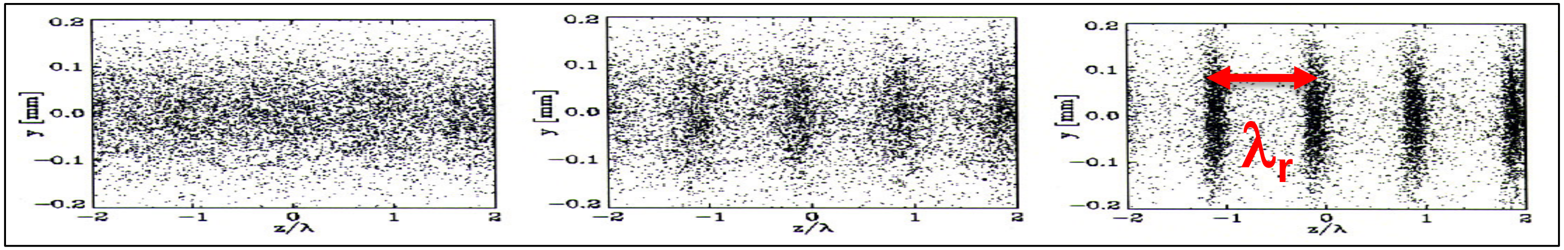


Free Electron Lasers

- Produced by **resonant interaction** of a relativistic electron beam with EM radiation in an undulator



$$\boxed{\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)}$$



- **Radiation intensity ∝ N^2**
- **Tunable, Powerful, Coherent radiation sources**

Self-Amplified Spontaneous Emission (SASE)

- Initiated by electron shot noise (spontaneous emission) and amplified over a narrow frequency bandwidth $\sigma_\omega \sim \rho\omega_r$

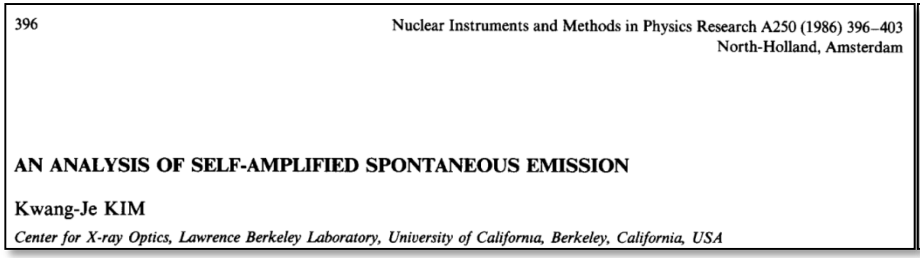
K.-J. Kim, NIMA (1986), Wang & Yu, NIMA (1986)

$$\frac{dP}{d\omega} = g_A\left(\frac{dP_0}{d\omega} + g_S\frac{\rho\gamma_0 mc^2}{2\pi}\right)\exp\left(\frac{z}{L_G} - \frac{\Delta\omega^2}{2\sigma_\omega^2}\right)$$

input power

effective start-up noise power ≈ undulator radiation over $2L_G$

- To determine the 3D effects including diffraction and finite beam size, one must solve the initial value problem in terms of a set of guided modes (first introduced by G. Moore)



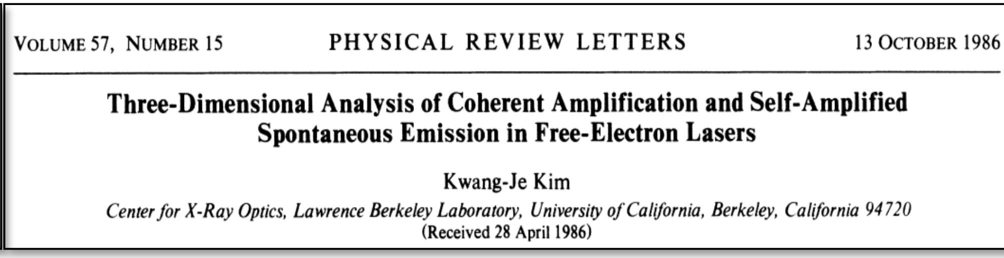
396

Nuclear Instruments and Methods in Physics Research A250 (1986) 396–403
North-Holland, Amsterdam

AN ANALYSIS OF SELF-AMPLIFIED SPONTANEOUS EMISSION

Kwang-Je KIM

Center for X-ray Optics, Lawrence Berkeley Laboratory, University of California, Berkeley, California, USA



VOLUME 57, NUMBER 15 PHYSICAL REVIEW LETTERS 13 OCTOBER 1986

Three-Dimensional Analysis of Coherent Amplification and Self-Amplified Spontaneous Emission in Free-Electron Lasers

Kwang-Je Kim

Center for X-Ray Optics, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 28 April 1986)

Vlasov-Maxwell formalism

- The interaction between the electron beam and the FEL radiation can be described in the framework of the Vlasov-Maxwell equations.

- The e-beam is described in terms of a distribution function $F = F(\theta, \eta, \boldsymbol{x}, \boldsymbol{p}; z)$ in 6D-phase space. In view of the importance of stochastic effects such as shot noise, we use the Klimontovich distribution:

$$F(\theta,\eta,\boldsymbol{x},\boldsymbol{p};z) = \frac{k_1}{n_e}\sum_{j=1}^{N_e}\delta[\theta-\theta_j(z)]\delta[\eta-\eta_j(z)] \times \delta[\boldsymbol{x}-\boldsymbol{x}_j(z)]\delta[\boldsymbol{p}-\boldsymbol{p}_j(z)],$$

n_e: on-axis electron number density

- The distribution function is governed by the Vlasov equation

$$\frac{\partial F}{\partial z} + \frac{d\theta}{dz}\frac{\partial F}{\partial \theta} + \frac{d\eta}{dz}\frac{\partial F}{\partial \eta} + \frac{d\boldsymbol{x}}{dz}\cdot\frac{\partial F}{\partial \boldsymbol{x}} + \frac{d\boldsymbol{p}}{dz}\cdot\frac{\partial F}{\partial \boldsymbol{p}} = 0,$$

K.-J. Kim, PRL 57, 1871 (1986)

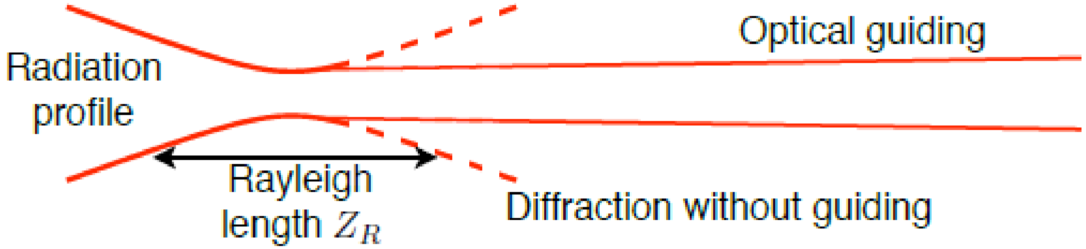
K.-J. Kim, Z. Huang, R. Lindberg, Synchrotron Radiation and FELs (Cambridge Press, 2017)

Van Kampen's normal mode expansion

- After linearizing Vlasov Eq., we seek the self-similar, guided eigenmodes of the FEL. These are solutions of the form:

$$\Psi = \begin{bmatrix} a_\nu(\hat{\boldsymbol{x}};\hat{z}) \\ f_\nu(\hat{\eta},\hat{\boldsymbol{x}},\hat{\boldsymbol{p}},\hat{z}) \end{bmatrix} = e^{-i\mu_\ell \hat{z}} \begin{bmatrix} \mathcal{A}_\ell(\hat{\boldsymbol{x}}) \\ \mathcal{F}_\ell(\hat{\boldsymbol{x}},\hat{\boldsymbol{p}},\hat{\eta}) \end{bmatrix}$$

- They are characterized by a constant growth rate μ_l and a z-independent radiation/density mode profile A_l/F_l (**Optical guiding**)



- Substituting into the Vlasov-Maxwell (FEL) equations, we obtain two coupled relations for the growth rate and the mode amplitudes:

$$\begin{bmatrix} \mu_\ell \mathcal{A}_\ell + \left(-\frac{\Delta\nu}{2\rho} + \frac{1}{2}\hat{\boldsymbol{\nabla}}_\perp^2\right)\mathcal{A}_\ell + i\int d\hat{\boldsymbol{p}}d\hat{\eta}\,\mathcal{F}_\ell \\ \mu_\ell \mathcal{F}_\ell + i\mathcal{A}_\ell \frac{\partial \bar{f}_0}{\partial \hat{\eta}} + \left\{-\nu\dot{\theta} + i\left(\hat{\boldsymbol{p}}\cdot\frac{\partial}{\partial \hat{\boldsymbol{x}}} - \hat{k}_\beta^2 \hat{\boldsymbol{x}}\cdot\frac{\partial}{\partial \hat{\boldsymbol{p}}}\right)\right\}\mathcal{F}_\ell \end{bmatrix} = 0.$$

3D solution

- Using Gaussian distributions, we obtain an explicit dispersion relation:

$$\left(\mu - \frac{\Delta\nu}{2\rho} + \frac{1}{2}\hat{\nabla}_{\perp}^{2}\right) A(\hat{x}) - \frac{1}{2\pi \hat{k}_{\beta}^{2} \hat{\sigma}_{x}^{2}} \int_{-\infty}^{0} d\tau \, \tau e^{-\hat{\sigma}_{\eta}^{2}\tau^{2}/2 - i\mu\tau}$$

$$\times \int d\hat{p} \, A[\hat{x}_{+}(\hat{x},\hat{p},\tau)] \exp\left[-\frac{1 + i\tau \hat{k}_{\beta}^{2}\hat{\sigma}_{x}^{2}}{2\hat{k}_{\beta}^{2}\hat{\sigma}_{x}^{2}} \left(\hat{p}^{2} + \hat{k}_{\beta}^{2}\hat{x}^{2}\right)\right] = 0.$$

- There are four dimensionless parameters that affect the growth rate

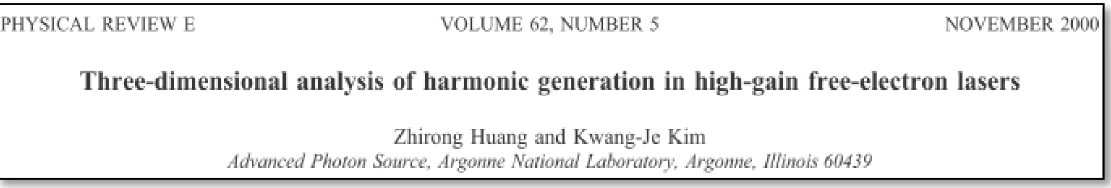
  (***L.-H. Yu, Krinsky, Gluckstern, Phy. Rev. Lett. 64, 1990***)

  - $\hat{\sigma}_x$ is a quantitative measure of the diffraction effect
  - $\hat{\sigma}_x \hat{k}_\beta$ is a measure of the emittance effect
  - $\hat{\sigma}_\eta$ represents the energy spread effect
  - $\Delta\nu/(2\rho)$ is scaled frequency detuning

- Ming Xie obtained a fitting formula that captures all these effects for FEL designs (1995)

Kwang-Je at APS since 1998

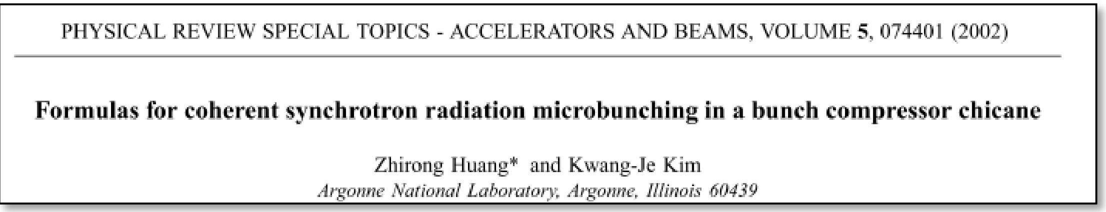
- Kwang-Je arrived at APS in 1998, I followed Kwang-Je to Chicago right after my Ph.D. from Stanford in May 1998.
- Our work was largely supported by an Argonne LDRD to do "***Comprehensive Analysis of SASE***".
- In 1998-2002 we studied short-pulse effects, harmonic generation, 3D SASE start-up, FEL saturation, CSR microbunching instability...
- Together we published >20 journal publications and numerous conference papers, and went to some nice workshops too!



PHYSICAL REVIEW E VOLUME 62, NUMBER 5 NOVEMBER 2000

Three-dimensional analysis of harmonic generation in high-gain free-electron lasers

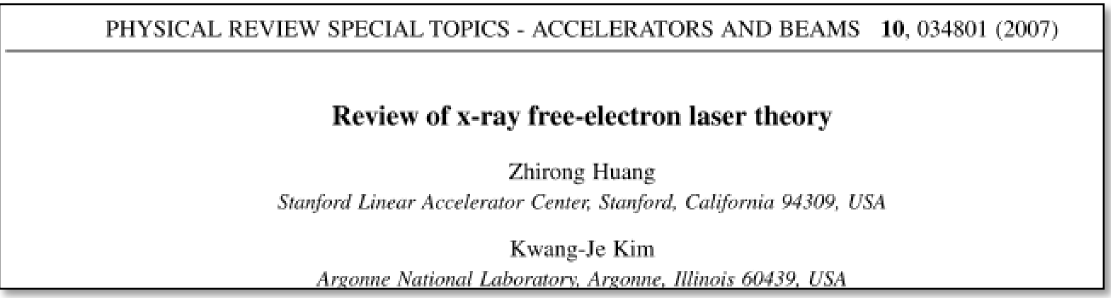
Zhirong Huang and Kwang-Je Kim
Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439



PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **5**, 074401 (2002)

Formulas for coherent synchrotron radiation microbunching in a bunch compressor chicane

Zhirong Huang* and Kwang-Je Kim
Argonne National Laboratory, Argonne, Illinois 60439



PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **10**, 034801 (2007)

Review of x-ray free-electron laser theory

Zhirong Huang
Stanford Linear Accelerator Center, Stanford, California 94309, USA

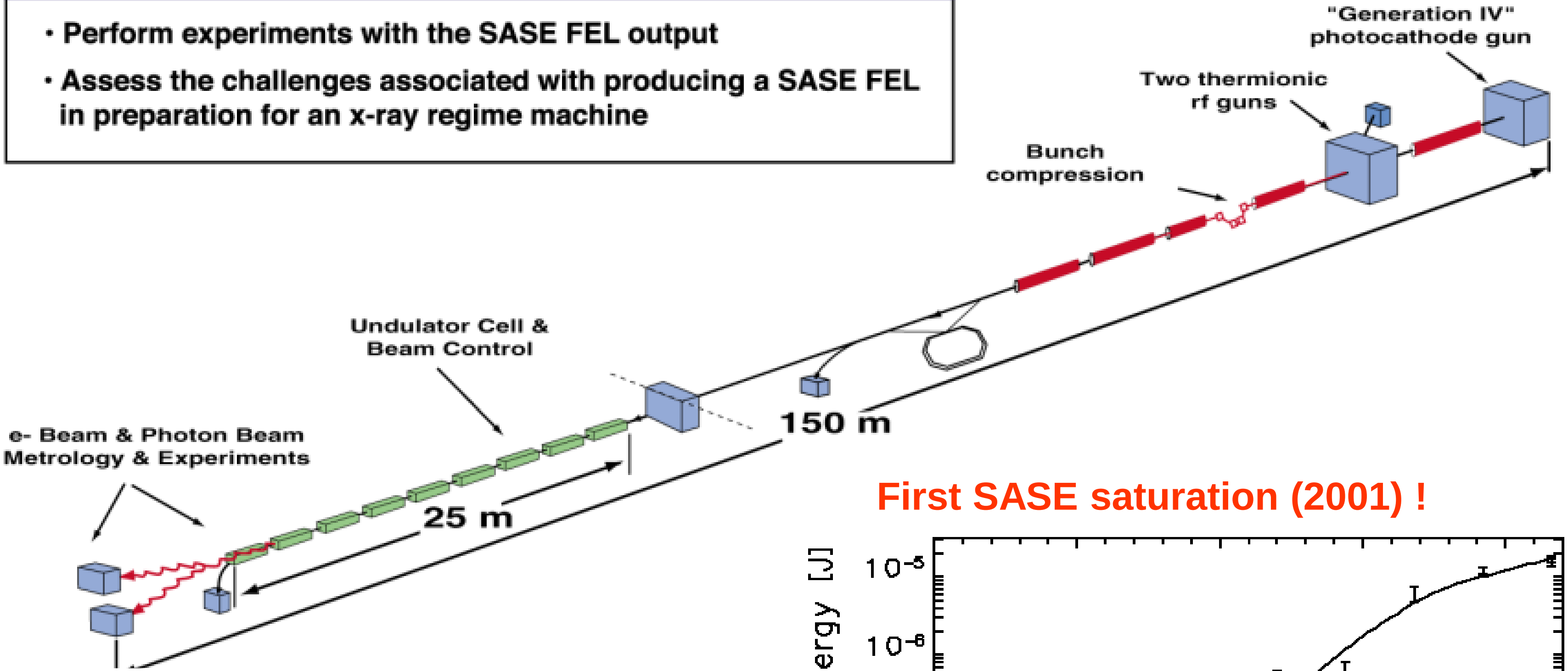
Kwang-Je Kim
Argonne National Laboratory, Argonne, Illinois 60439, USA

Sardinia beach (Italy 2002)

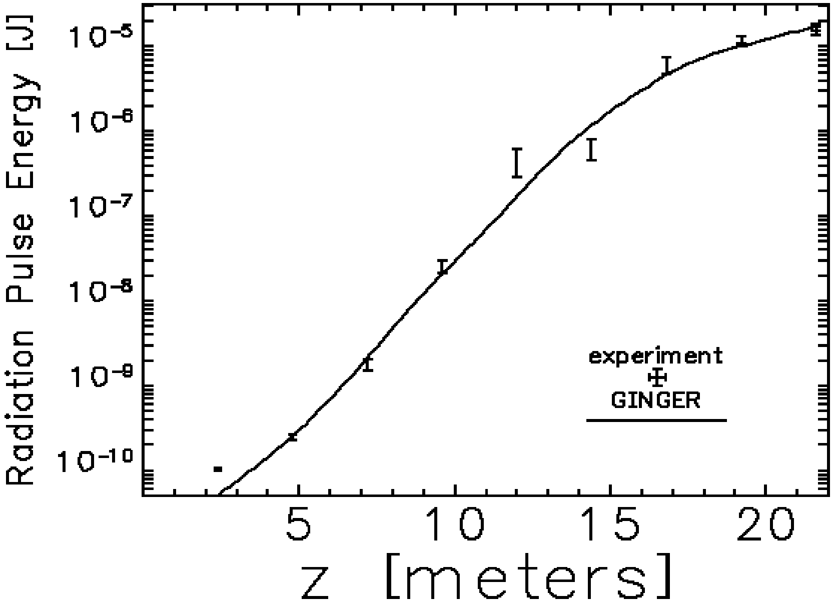
LOW-ENERGY UNDULATOR TEST LINE PARAMETERS

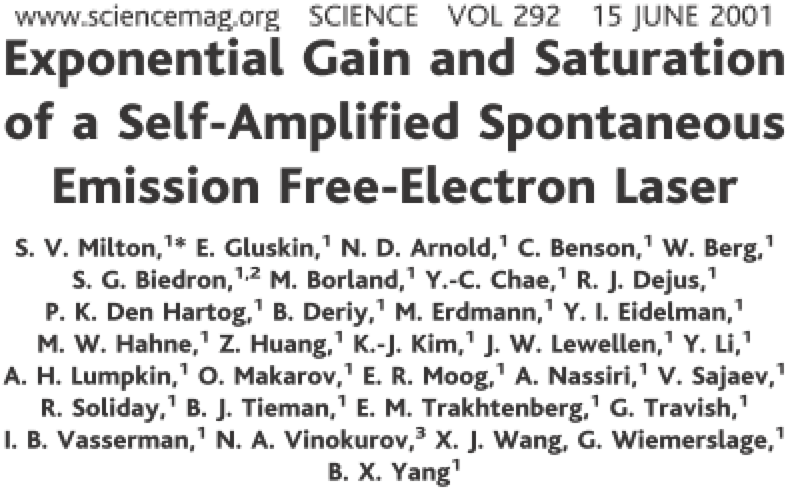
PROJECT GOALS

- Perform experiments with the SASE FEL output
- Assess the challenges associated with producing a SASE FEL in preparation for an x-ray regime machine



First SASE saturation (2001) !





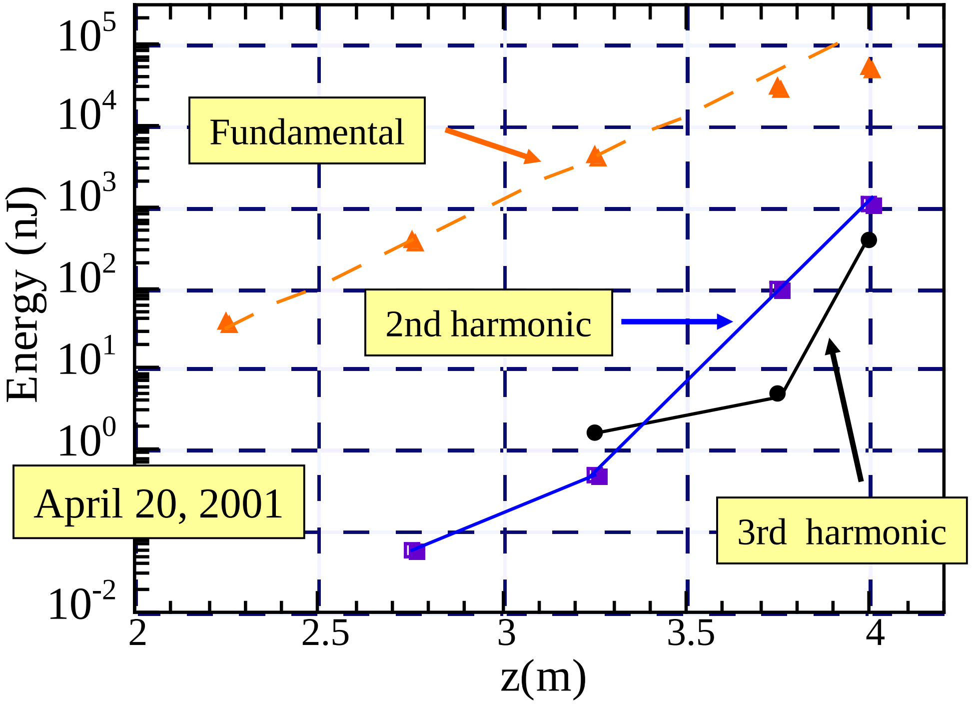
www.sciencemag.org SCIENCE VOL 292 15 JUNE 2001

Exponential Gain and Saturation of a Self-Amplified Spontaneous Emission Free-Electron Laser

S. V. Milton,[1*] E. Gluskin,[1] N. D. Arnold,[1] C. Benson,[1] W. Berg,[1]
S. G. Biedron,[1,2] M. Borland,[1] Y.-C. Chae,[1] R. J. Dejus,[1]
P. K. Den Hartog,[1] B. Deriy,[1] M. Erdmann,[1] Y. I. Eidelman,[1]
M. W. Hahne,[1] Z. Huang,[1] K.-J. Kim,[1] J. W. Lewellen,[1] Y. Li,[1]
A. H. Lumpkin,[1] O. Makarov,[1] E. R. Moog,[1] A. Nassiri,[1] V. Sajaev,[1]
R. Soliday,[1] B. J. Tieman,[1] E. M. Trakhtenberg,[1] G. Travish,[1]
I. B. Vasserman,[1] N. A. Vinokurov,[3] X. J. Wang, G. Wiemerslage,[1]
B. X. Yang[1]

Nonlinear Harmonic Radiation at VISA*

Nonlinear Harmonic Energy vs. Distance



Energy Comparison

Mode (n)	Wavelength (nm)	Energy (µJ)	% of E_1
1	845	52	
2	421	.93	1.8
3	280	.40	.77

Using the relation of 2nd and 3rd harmonic energies as given by Z. Huang and K.J.Kim

$$E_2 = \left(\frac{K}{\gamma k_u \sigma_x}\right)^2 \left(\frac{K_2}{K_3}\right)^2 \left(\frac{b_2}{b_3}\right)^2 E_3$$

b -bunching parameters
K_n -Coupling coefficients

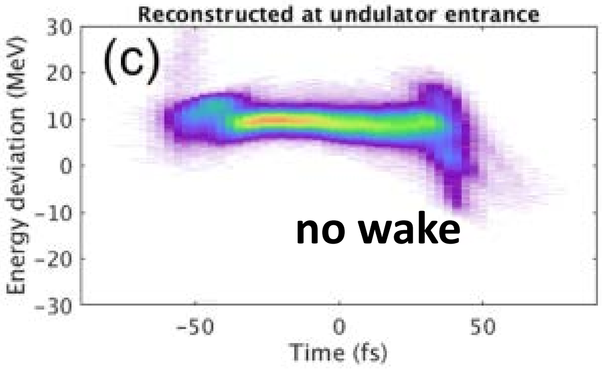
Associated gain lengths

$$L_f = 19cm$$
$$L_2 = 9.8cm \quad \Longrightarrow \quad L_n = L_g / n$$
$$L_3 = 6.0cm$$

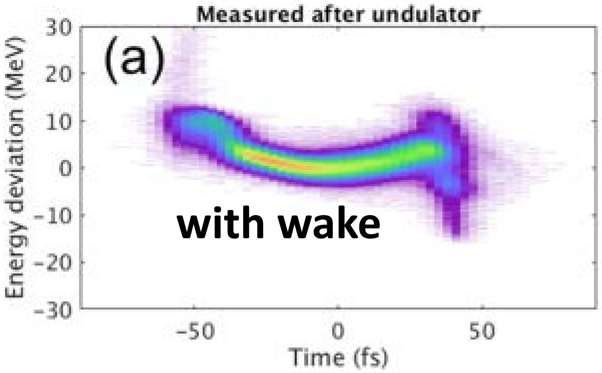
* A. Tremaine, XJ Wang et al., PRL (2002)

Onto LCLS

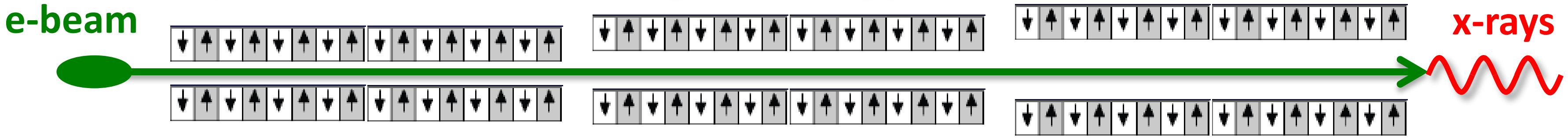
- I left Chicago for the Sunny California in late 2002.
- It was realized that undulator wakefield-induced energy loss is an important effect for the LCLS (5 mm gap for >100 m)



With XTCAV
(Y. Ding)



- Compensate the average energy loss by tapering undulator



- Tapered undulator keeps FEL resonance and increase power.
- But, undulator wakefield makes time-dependent energy loss and hence taper only works for the average loss.
- FEL resonance cannot be kept for every slice of the bunch.
- This led to FEL power degradation.

FEL with slowly varying beam and undulator parameters

- E-beam energy $\gamma_c(z)$, undulator parameter K(z)
- Initial resonant wavelength $\lambda_0 = \frac{2\pi}{k_0} = \frac{\lambda_u}{2\gamma_c(0)^2}\left[1 + \frac{K(0)^2}{2}\right]$
- Resonant energy $\gamma_r(z) = \sqrt{\frac{\lambda_u}{2\lambda_0}\left[1 + \frac{K(z)^2}{2}\right]}$
- Longitudinal motion is described by

$\theta = (k_0 + k_u)z - ck_0t^*$ (ponderomotive phase)

$\eta = \frac{\gamma(z) - \gamma_c(z)}{\rho\gamma_c(0)}$ (normalized energy, change only due to FEL)

$$\boxed{\begin{aligned} \frac{d\theta}{dz} &= 2k_u\frac{\gamma(z)-\gamma_r(z)}{\gamma_c(0)} = 2k_u\rho\left[\eta + \frac{\gamma_c(z)-\gamma_r(z)}{\rho\gamma_c(0)}\right] \\ \frac{d\eta}{dz} &\propto E\cos(\theta + \phi) \end{aligned}}$$

(E and ϕ are radiation field and phase)

- *Z. Huang, G. Stupakov, Phys. Rev. ST Accel. Beams 8, 040702 (2005)*

WKB approximation

- Well-known technique in QM for slowly-varying potential
- FEL is characterized by ρ: the relative gain bandwidth is a few ρ, and radiation field gain length ~ $\lambda_u/(4\pi\rho)$
- Relative change in beam energy w.r.t resonant energy

$$\delta(z) = \frac{1}{\rho}\frac{\gamma_c(z)-\gamma_r(z)}{\gamma_c(0)}$$

Normalized to ρ

- Apply WKB technique if the relative energy change per field gain length is smaller than ρ, i.e.,

$$\boxed{\left|\frac{d\delta}{d\tau}\right| < 1\,, \quad \tau = 2\rho k_u z = \frac{z}{\lambda_u/(4\pi\rho)}}$$

- We then extend the WKB analysis to 3D via Van Kampen’s method of mode expansion.

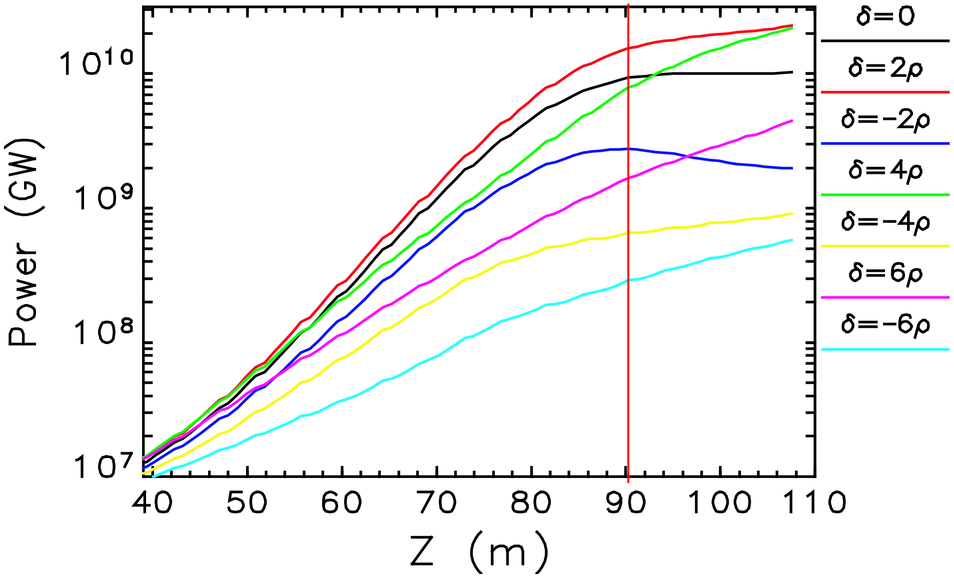
Comparison w/ simulations

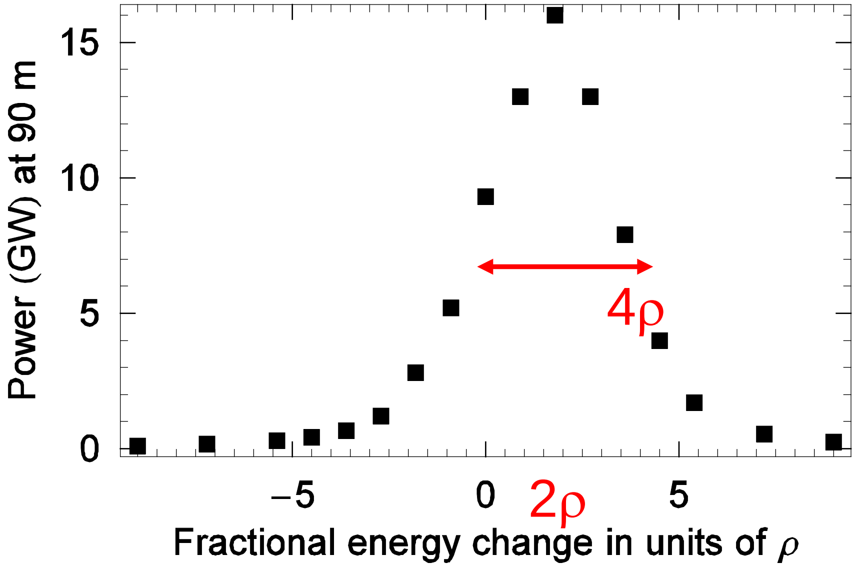
- ➢ Radiation power dependence on δ is a gaussian

$$\boxed{P(\delta;z) = P_m(z)\exp\left[-\frac{1}{2}\left(\frac{\delta(z)-\delta_m(z)}{\sqrt{3}\sigma_\omega/\rho}\right)^2\right]}$$

- ➢ *GENESIS* simulation of LCLS power vs. δ,

➔ Power enhancement ~ 2 when energy gain 2ρ at saturation



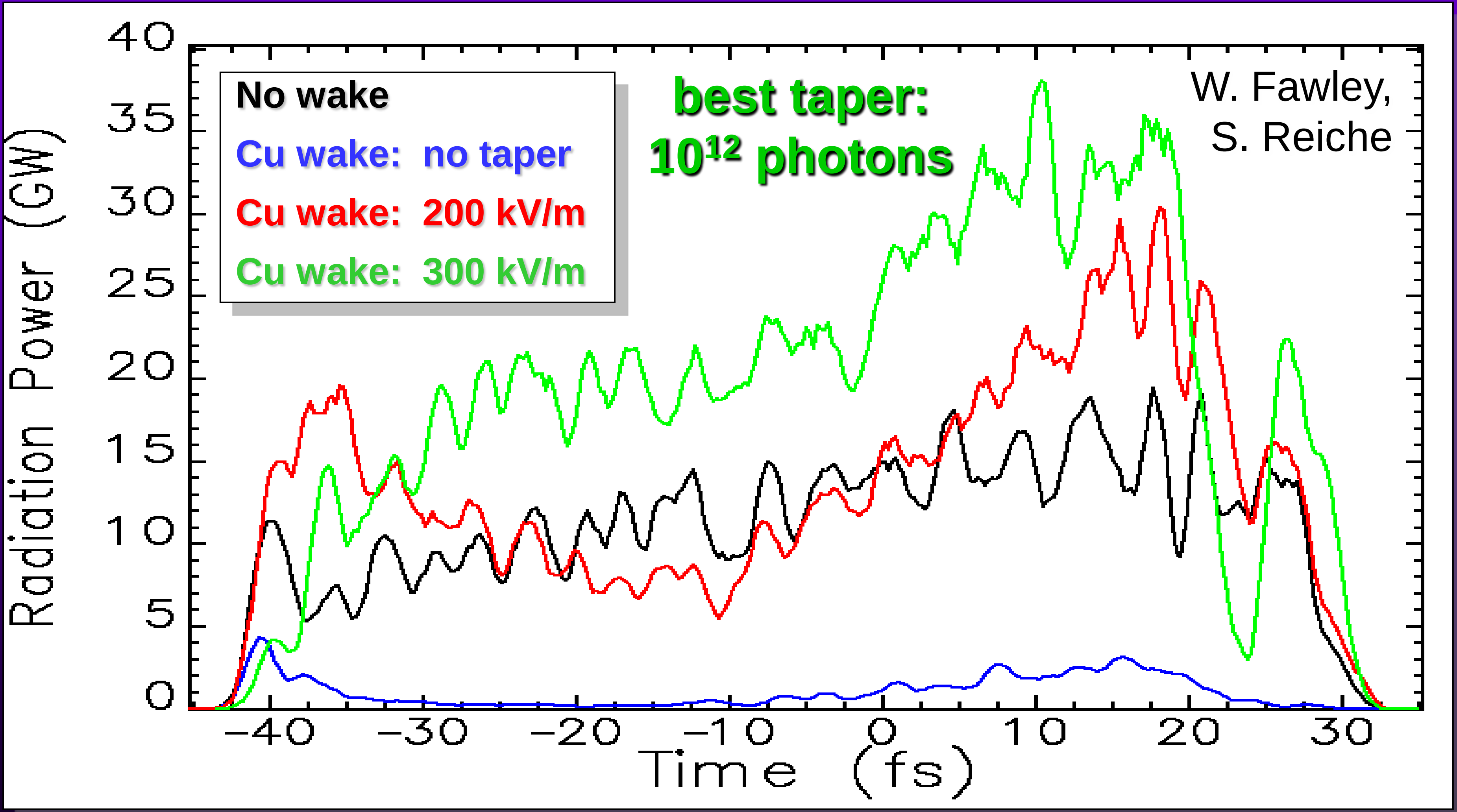


- ➢ Power vs. δρ has RMS $\sqrt{3}\sigma_\omega$

FWHM $4\sigma_\omega$ (~4ρ at saturation)

0.2-nC FEL Simulations with Taper

P. Emma's talk at PAC05

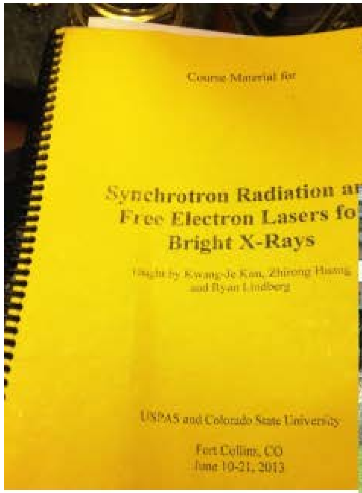


This study led to abandoning 1-nC LCLS

Teach FEL theory in USPAS

- ➢ KJK and I started the USPAS teaching in 2000, later joined force with Ryan. In total we have taught 8 USPAS sessions (+1 this coming summer).

- ➢ The lecture notes were steadily improved and became a textbook published by Cambridge Press in 2017.



KJ

Course Material for

Synchrotron Radiation a
Free Electron Lasers fo
Bright X-Rays

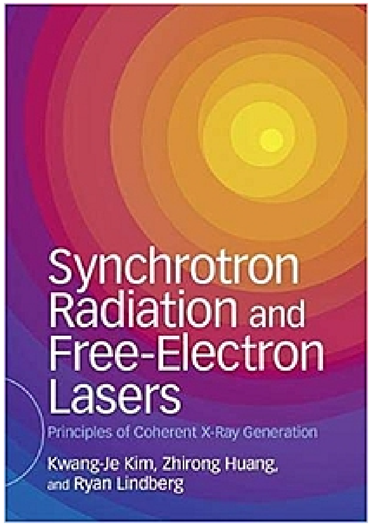
Taught by Kwang-Je Kim, Zhirong Huang
and Ryan Lindberg

USPAS and Colorado State University

Fort Collins, CO
June 10-21, 2013

Lecture notes 2013

2008 USPAS at Santa Rosa, CA



Synchrotron Radiation and Free-Electron Lasers

Principles of Coherent X-Ray Generation

Kwang-Je Kim, Zhirong Huang, and Ryan Lindberg

国家出版基金项目

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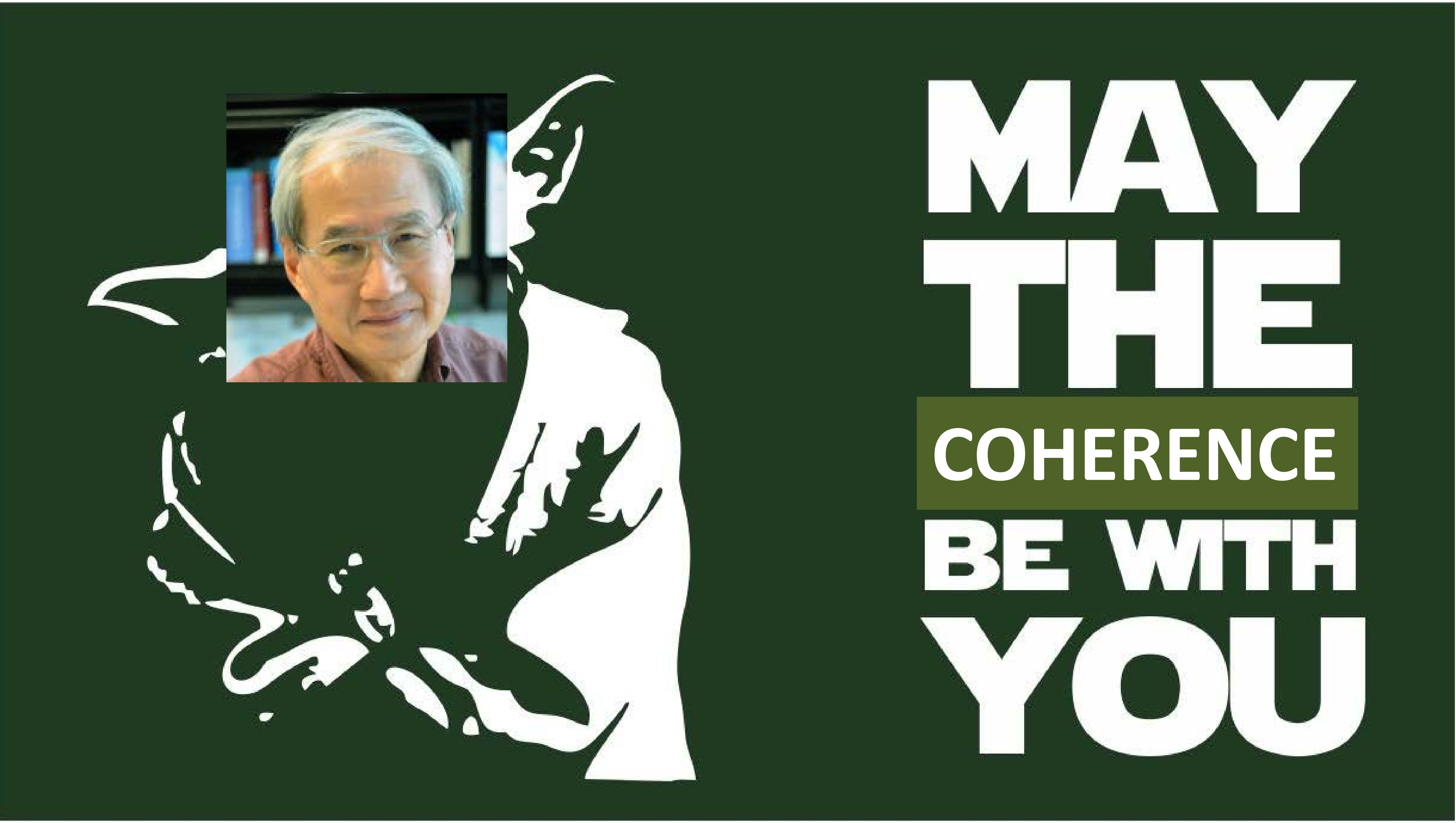
同步辐射与自由电子激光
——相干X射线产生原理

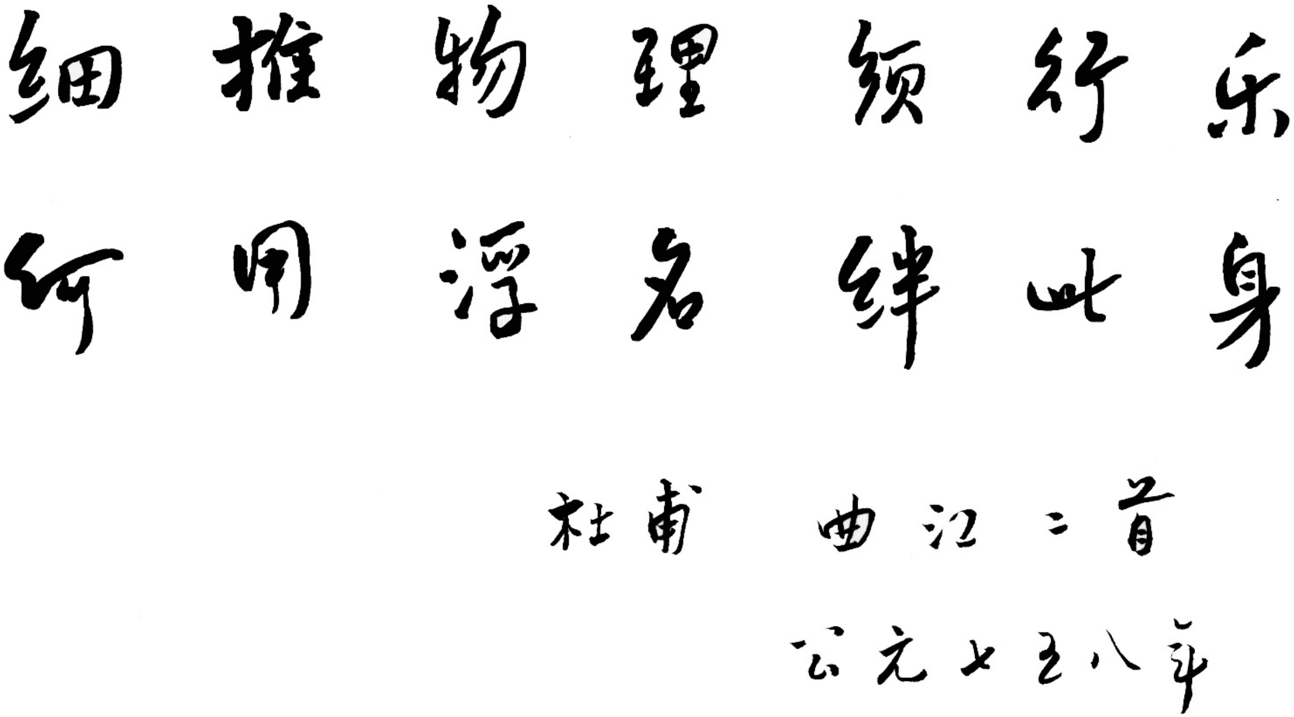
Synchrotron Radiation and Free-Electron Lasers: Principles of Coherent X-Ray Generation

[美]金光齐(Kwang-Je Kim)
[美]黄志戎
[美]瑞安·林德伯格(Ryan Lindberg) 著
黄森林 刘克新 译
黄志戎 审校

- ➢ The book is translated into Chinese in 2018.

(Kwang-Je Kim -> 金光齐 -> Coherent radiation)


MAY
THE
COHERENCE
BE WITH
YOU



细推物理须行乐
何用浮名绊此身

杜甫 曲江二首
公元七五八年

The law of Nature tells us to enjoy as we may.
Why spoil our joy by sheer vanity of life?

Poem by Fu Du (Tang dynasty, 758)
Calligraphy by T.-D. Lee (Nobel Laureate 1957)